Different forms of mathematical questions for different purposes: Comparing student responses to similar closed and open-ended questions

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Responses of Year 8 students to open-ended and closed mathematics questions, which addressed similar aspects of the curriculum, were compared. The responses of the students were examined and the elements of the tasks were listed. In two out of three pairs examined, the closed question was easier for students. In the other, the open-ended task was easier. It seems that closed and open-ended items may contribute productively both to classroom and specific assessment tasks, although their contribution may be different.

In the past, one of the main roles of mathematics has been as a filter of students wishing to proceed to tertiary studies. The curriculum reflected this orientation, and it was not expected that all students would study mathematics at senior levels nor that all those who did so would master the content. This orientation has now changed. Many more students are choosing to study mathematics at upper secondary levels and many tertiary courses now require an understanding of advanced aspects of secondary mathematics. There is also an expectation by employers, tertiary institutions and the community that graduates are able to use their mathematics in more diverse situations. Mathematics teachers are now striving to identify approaches which will allow them to assist all students to achieve success in mathematics and to provide them with the mathematical background which is necessary for their future studies and careers. Specifically, teachers are now seeking strategies, which allow more students to learn mathematics at higher levels, and which also stimulate ability to solve problems and apply mathematics in unfamiliar contexts.

Teachers are faced with a tension between, on one hand, posing tasks which provide opportunities for development of problem solving abilities and, on the other hand, addressing the content specified within syllabus documents and required for competitive examinations. One approach used to address these dual purposes is the use of content-specific open-ended questions (e.g. Sullivan, Clarke, & Wallbridge, 1991; Sullivan & Clarke, 1991; Sullivan, Bourke & Scott, 1995).

We use the term closed to refer to questions for which there is only one correct response. This is consistent with the approach of Pehkonen (1997). Note that some authors restrict the use of the term closed to multi-choice type items. In our terms, an example of a closed question is:

Find the mean of 8, 10, 12, 12, and 18.

The corresponding open-ended question would be:

The mean of a set of 5 scores is 12. What might be the scores?

The open-ended question draws on the same content, but allows the possibility of the students investigating the situation for themselves and so coming to a better appreciation of the concept of mean as a result of their own thinking. It also produces quite different classroom interactions in that the students can report on their own insights and the variety of solutions they find. With the closed questions, their responses are either right or wrong. There are also concerns with regard to using closed questions for assessment purposes.

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For example, Ellerton and Clements (1997), when using a short-answer (multiple-choice) pen and paper test found a 28% gap between student performance and student understanding. Students who lacked adequate understanding of the key concepts being tested gave correct answers, or students with full or partial understanding gave incorrect answers. These validity concerns support the investigation of the use of open-ended questions for assessment.

Curriculum developers have recently endorsed the use of open-ended questions. For example, the NSW Board of Studies (1996) provided teachers with many examples of sample open-ended questions, and such questions are now used in published curriculum materials (e.g., Sattler & White, 1997). This project seeks to examine the characteristics and effectiveness of different types of questions. Specifically it attempts to examine:

Are content-specific open-ended questions:

i) easier or more difficult than comparable closed questions?

ii) more likely to elicit higher level responses if the prompt is phrased within a practical context?

iii) more likely to elicit higher level responses if explicit prompts for multiple responses are used?

iv) able to be scored validly and reliably if used within an assessment context?

This paper focuses on the first of these questions. Content-specific items used in the project involved area and perimeter and decimals, although only data for area and perimeter are reported at this time.

The approach to data collection

Responses were sought from approximately 1200 students from schools in each of the three Australian states. Data were also collected in Indonesia but these are not reported here. In each case more than two schools responded to the tests. Students were selected to maximise comparability by gender mix, socio-economic status, urban location, school size, and experience of the teachers.

Eight instruments requiring written responses were developed. Each differed according to the mix of open-ended/closed and context/context free. The test forms also sought information on the use of appropriate cueing for multiple responses, the impact of the use of contexts in the phrasing of the questions, and the potential of a mix of open-ended and closed questions on assessment tasks scored holistically. Where possible the items were designed to be comparable with items on large-scale tests used elsewhere. The items were piloted in three states to ensure that there were no difficulties with the wording, and that the content was compatible with the syllabus.

The items on each of the tests were scored individually. The closed items were scored as either correct or incorrect. Each of the open-ended items were scored using the following codes:

i one or two correct responses
ii some correct, some incorrect
iii three or more correct responses

The various forms of the tests were administered to parallel streams across schools and states. At each of the schools the tests were given on the same day.

Comparing responses of students to open-ended and closed questions

For this paper three pairs of open-ended and closed items are compared. The same group of students answered each pair of items. The open-ended items used a prompt "give at least three answers" since during trialling we observed that many students were reluctant to give more than one response. The purpose of each pair of questions was first, to gauge the effect of using differing units of measure, second, to ascertain children's' understanding of the interrelationship between perimeter and area, and third to determine children's' understanding of the relationship between embedded rectangles

Different units

The items requiring the use of different units were designed to be comparable to the following item taken from the Department of Education (1991).

A rectangular rug has an area of 2 square metres. The rug is 40 centimetres wide. How long would it be?

5 m *8 m* *20 m* *50 m*

The Department of Education (1991) study reported that 52% percent of year 9 students were able to answer this correctly. The results for the present study are shown in Table 1.

Table 1: Responses to *different unit* items

Item	Type	Response Code	% of correct responses	n
This rectangle has an area of $2m^2$. It is 40cm wide. How long is it?	Closed	Correct	7	226
A rectangle has an area of $3m^2$. What might be the length and width of the rectangle? (give at least three answers)	Open-ended	i	8	226
		ii	6	
		iii	9	

Given the facility of the Department of Education (1991) question, the number of correct responses to the closed item (7%) seemed very low. This may be due to the younger year level used for the sample in this project or to the multi-choice selection of responses given in the original question. The multiple choice format for the Department of Education (1991) supports the hypothesis that a number of students simply had to choose between *5m* and *50m,* and thus did not really understand the units involved. This supports the Ellerton and Clements (1997) finding.

A total of 23% of students gave one or more correct responses to the open-ended item. It appears from the results that the open-ended item may be easier than the closed item. An examination of the types of responses for each item gives some insight into why the open-ended question seemed easier. The distribution of responses were as follows:

For the Closed item:

7 students converted to 0.4m then calculated correctly
4 students converted to 20 000cm^2
5 students gave the correct answer although the working out seemed incorrect.

For the Open-ended item:

52 students gave one or more correct responses
48 of these gave 3m x 1m as one of their answers
21 gave 1.5m x 2m as one of their responses
most other correct responses used decimals in some way
another 120 responses were correct numerically but with incorrect units

The percentage of students in the last category of the open-ended question (53%) is comparable with the percentage of correct responses to the Department of Education (1991) question, attesting that the need to use compatible units of measure was the main source of difficulty that students experienced. The results indicate that the students experienced little difficulty with the concept of multiple responses. Even though there were still comparatively few responses to the open-ended item, there were more students who were able to explore the context with the open-ended prompt, with many of these going beyond the trivial 3 x 1 response.

Based on these responses, and on an analysis of the questions, each item was broken into the components needed for success on each. Figure 1 summarises the components. The components common to both items are recorded in the middle column of the table.

For Closed item	For Both	For Open-ended item
Match information to dimensions	Read	Select 2 numbers with product of 3
Convert m^2 to cm^2 OR cm to m	Comprehend	Recognise multiple possibilities
Do division	Encode	
	Record	
	Area is L x W	

Figure 1: Components for each item.

For these two problems, it is possible that the closed item is more difficult because of the need both to recognise the need to convert from one unit to the other and then to do just that. Problems with the units may impede students from exhibiting their understanding of the area concept. The open-ended item does not necessarily require unit conversion, and it seems that those students who responded correctly did so without converting. Hence, the open-ended item gave more information about student's understanding of area but gave little information about their facility with unit conversion. The use of an appropriate prompt may be needed to elicit this information.

In the classroom, the open-ended item may be useful for initial exploration of the area concept with either non whole number sides or mixed units, and the relationship between differing units of measure.

Perimeter and Area

The items related to Perimeter and Area were designed to build on the following item taken from the Department of Education (1991).

What is the area of a rectangle 8 metres long and 3 metres wide?

In that report, 67% of the year 7 students responded correctly.

These items were also related to the following item from Sullivan & Clarke (1991).

A rectangle has a perimeter of 30m. What might be the area?

In that study, 31% of year 6 students and 80% of year 10 students gave one or more correct responses. The pair of items used for this study is shown in Table 2:

Table 2: Responses to *perimeter and area* items

Item	Type	Response Code	% of correct responses	n
A rectangle is 8m long and 5m wide. What is the perimeter and area of the rectangle?	Closed	Perimeter Area	87 67	137
If a rectangle has a perimeter of 30m, what might be the area of the rectangle? (Give at least 3 answers)	Open-ended	i ii iii	9 4 43	137

In this case the closed item elicited more correct responses. Most students also exhibited a higher facility in finding the perimeter of a rectangle as compared with finding the area of the rectangle. This was consistent across all states and schools. The area response was similar to the item. For the open-ended question, 56% of the sample could offer one or more correct answer. The following shows a detailed breakdown of the responses given.

For the closed item:

For perimeter

- 49 students used 5 + 5 + 8 + 8
- 38 students used (5 x 2) + (8 x 2)
- 1 student used (5 + 8) x 2
- 32 students were scored correct but did not show working

For area

- 45 students used 5 x 8 with correct units
- 22 students used 5 x 8 by did not specify units (and were scored as correct)
- 25 students were scored as correct but did not show working

For the open-ended item:

2 students (only) used 7.5 x 7.5
all other correct responses used one or more of the whole number possibilities

Again students avoided the use of non-whole number units of measure. The components of the respective items are shown in Figure 2.

For Closed item	For Both	For Open-ended item
	Read	P and A are linked
	Comprehend	Reverse P calculation
	Know P	Do 1 calculation
	Know A	Recognise alternative possibilities
	Calculate	Further calculations
	Record	

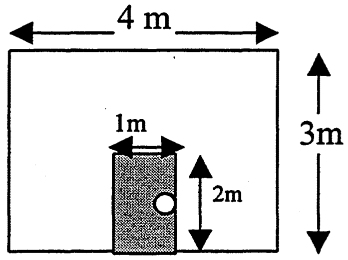
Figure 2: Components of the different items.

The open-ended item requires more integration of ideas (e.g. perimeter and area are linked) which could account for its lower facility. Perhaps in this case, the open-ended item would be better used after students have had experience with a variety of closed items. However, note that the skills required for the open-ended item are desirable and that the open-ended item provides the opportunity to build effective links between units, area and perimeter. Asking for (say) 5 answers could have better assessed the existence of links with units so that fractional lengths would be required.

Embedded rectangles

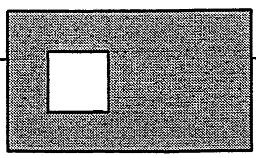
The items related to embedded rectangles were designed to build on the following item taken from the Department of Education (1991).

I want to paint the wall but not the door. What area of wall do I need to paint?



In that report, 49% of the year 7 students responded correctly. The results for this study are shown in Table 3.

Table 3: Responses to *embedded rectangles* items

Item	Type	Response Code	% of responses	n
What is the area of the shaded part of the rectangle? (same diagram as above – but with rectangle and not the door shaded)	Closed	Correct	77	216
The area of the shaded part of this diagram is $8m^2$. What might be the dimensions of the large and small rectangles (Give at least 3 answers) 	Open-ended	i ii iii	12 15 24	216

The above results indicate that again students gave more correct responses to the closed item. This was consistent across all states and schools. The distribution of responses to the open-ended and closed questions is shown below.

For the closed item:

136 students used (3 x 4) – (1 x 2) using correct units
17 students gave a correct answer with no working out shown
11 students gave the correct number but used no units
2 gave the correct answer, but seemed to have incorrect working

For the open-ended item:

260 correct responses used whole numbers dimensions
19 correct responses used decimals
9 responses were partially correct
45 students made no attempt

Twenty percent of students gave no response to the open-ended item. The preference for whole number units is again evident. The components for the respective items are presented in Figure 3.

For Closed item	For Both	For Open-ended item
	Calculate area	See need for trial and error
	Recognise shape as one rectangle taken away from another	Recognise multiple possibilities

Figure 3: Components of the *embedded rectangle* items.

As with the preceding pair of questions, the open-ended item appears to have some additional complexity. The 20% of non-attempts for the open-ended version would seem to suggest that the task of subtraction of areas is an obstacle. However, the high facility for the closed item suggests that this does not seem to have been a major problem. For the open-ended item, it may be that the mental effort required for using trial and error to find correct combinations could be a barrier to some students.

Conclusion

The results from this research give some insights the use of open-ended and closed questions. In two out of the three pairs students found the open-ended questions more difficult. An analysis of the types of responses, and summary of the components of the tasks, suggested that the second and third open-ended questions required thinking above and beyond that required for the corresponding closed question. The nature of this thinking needs to be further explored, as it may be that this type of thinking is what we value most in mathematics. Those open-ended items also required students to understand the links between two concepts and to be able to use such links to conjecture and generalise. It may also be that such questions are more appropriate to use once students have some understanding of the concepts involved, including experience with similar closed questions. The open-ended questions could then serve the role of stimulating students' thinking to higher levels. It also seems that an analysis of the components of similar tasks can indicate their comparative difficulty.

Two ancillary results were also interesting. Students seem to be more able to answer perimeter questions as compared with area questions. At Year 8 it seems that most students would have been exposed to both concepts. That one fifth of students could solve the perimeter and not area item is a puzzle.

Second, the use of mixed metric units in the problem seems to make it more difficult to answer. The facility for that item was very low. Perhaps students are seldom exposed to questions for which they have to combine two different aspects of the curriculum.

As opposed to the closed questions, which often show what students do not know, responses which contained correct and incorrect answers are useful in assessing both what students can do and what they cannot do. However, only correct responses to open-ended questions give insights into what students know but may not show what they do not know. Appropriate prompts may be needed to elicit this information.

These results overall will contribute to our understanding of the ways students approach such questions, and how open-ended questions can contribute to a broader assessment routine.

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